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DYNAMIC SAM ENDGAME MODEL

ANALYST'S MANUAL

N62269-76-C-0386

SEPTEMBER 1977

Kearney: Management Consultants

CAYWOOD-SCHILLER DIVISION
A. T. KEARNEY, INC.
100 South Wacker Drive
Chicago, Illinois 60606

DYNAMIC SAM ENDGAME MODEL

ANALYST'S MANUAL

FINAL REPORT

Under

Contract N62269-76-C-0386

SEPTEMBER 1977

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A model for evaluating kill probabilities associated with trajectories and intercept conditions generated by the NADC Dynamic SAM Model is described.

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FOREWORD

The activities described in this report in modifying the Dynamic SAM Model to include an endgame capability were performed during the period July 1976 to September 1977 under Contract N62269-76-C-0386 for the Naval Air Development Center, Warminster, Pennsylvania. The purpose of this effort was to develop a means for evaluating kill probabilities associated with trajectories and missile/aircraft intercept conditions generated by the NADC Dynamic SAM Model. This is the first of two volumes constituting the final report. The work was performed by R. H. Rose, M. A. Dloogatch, and D. S. Kluk of the Caywood-Schiller Division of A. T. Kearney.

This volume includes a summary of the work, and an analytic description of the model.

1. INTRODUCTION

The Dynamic SAM-Aircraft Model (DSAMAM) was originally designed and implemented by Autonetics in 1969. The intent was to provide a tool for evaluating the effectiveness of a maneuvering aircraft versus a command guided surface-to-air missile. The output of this model consisted of the intercept geometry for each missile. No attempt was made to determine a fuzing or detonation point or a kill probability.

This original model was programmed in FORTRAN IV to run on an IBM 360 computer. At some later date the necessary changes were made to allow the model to be run on the CDC computer at NADC. At this time a modification was made so that a kill probability was calculated based on the assumption of detonation at the point of closest approach. This PK was based on aircraft lethal radius inputs and a specified CEP for the missile.

The objective of the current study was to provide DSAMAM the capability of simulating the fuzing characteristics and terminal effects of both the SA-2 and SA-3 proximity-fuzed warheads.

Several modular subroutines were built into DSAMAM in order to complete this objective. These allow for a conical-pattern radar fuze, a backup contact fuze, calculation of blast and fragment kill for each component of the aircraft, and a cumulative kill probability for the entire aircraft.

2. DETAILED DESCRIPTION OF ENDGAME ADDITIONS TO DSAMAM

The following description assumes that the reader is familiar with Volumes I-IV of the Final Report on the Dynamic SAM-Aircraft Study, Report No. C9-2336/120, Autonetics, North American Rockwell Corporation.

2.1 Logical Flow

The flow chart of Figure 2-1 depicts the logic involved in determining whether or not fuzing has occurred at each time step, and if it has, calculating kill probabilities for each component. The notation used in these flow charts is as follows:

DHK	- direct hit with kill
DHNK	- direct hit without kill
$P_D(CF)$	- probability that contact fuze is dud
$P_D(RF)$	- probability that radar fuze is dud
$T_B(CF)$	- time of detonation due to contact fuze
$T_B(RF)$	- time of detonation due to radar fuze
T(DHK)	- time at which direct hit with kill occurs
T(RF)	- time at which radar fuzing occurs
T(DHNK)	- time at which direct hit without kill occurs
$P_K(T)$	- probability of component kill due to fragment spray given detonation at time T
CONFUZE	- subroutine which determines whether or not the conical radar fuze has been activated
SPRAY	- subroutine which calculates the component kill probabilities resulting from fragment damage due to an explosion at some specified time
STAB	- subroutine which determines whether any of a set of direct-hit ellipsoids has been pierced by the path of the missile during the current time step

FIGURE 2-1
LOGIC FOR PK DETERMINATION

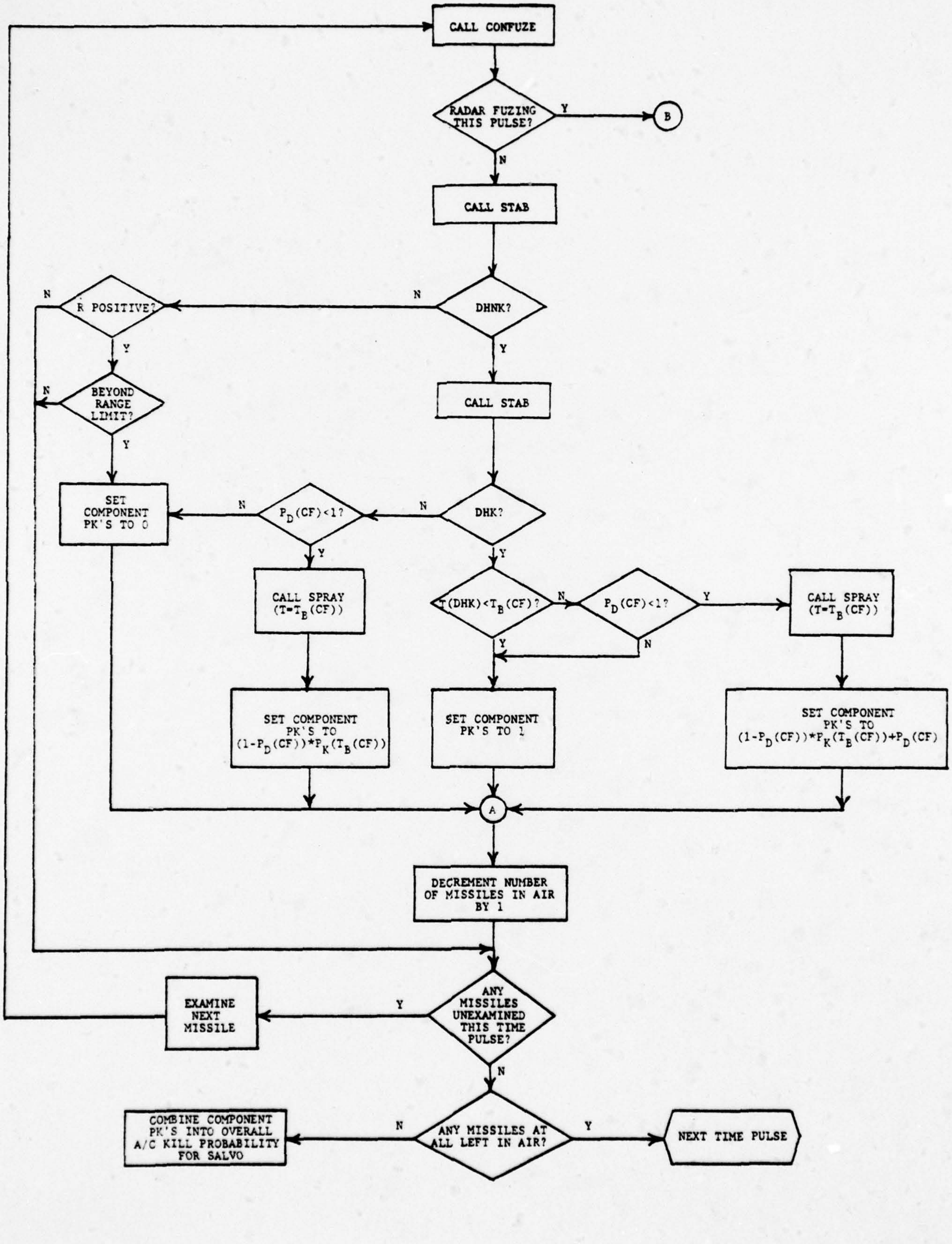
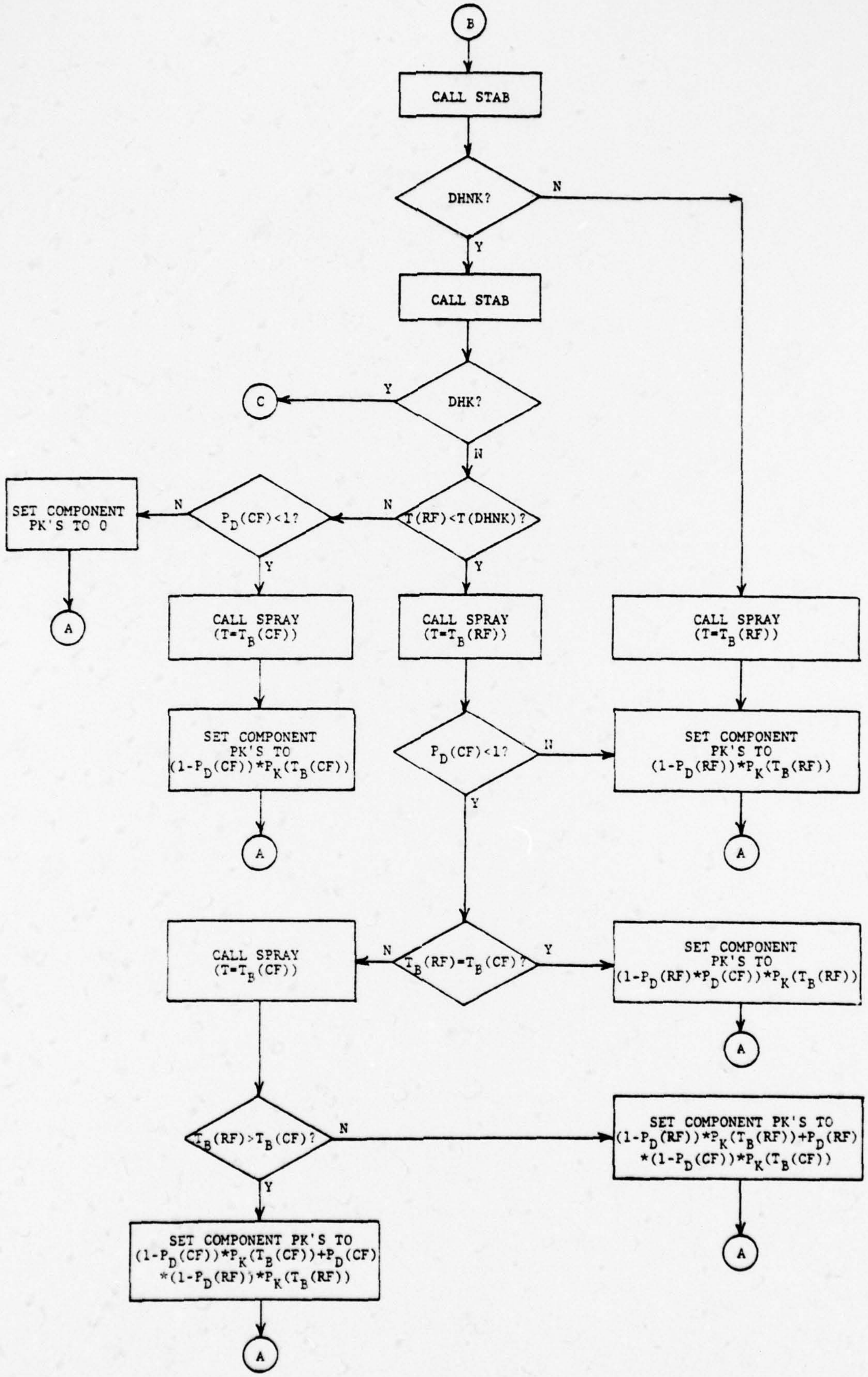
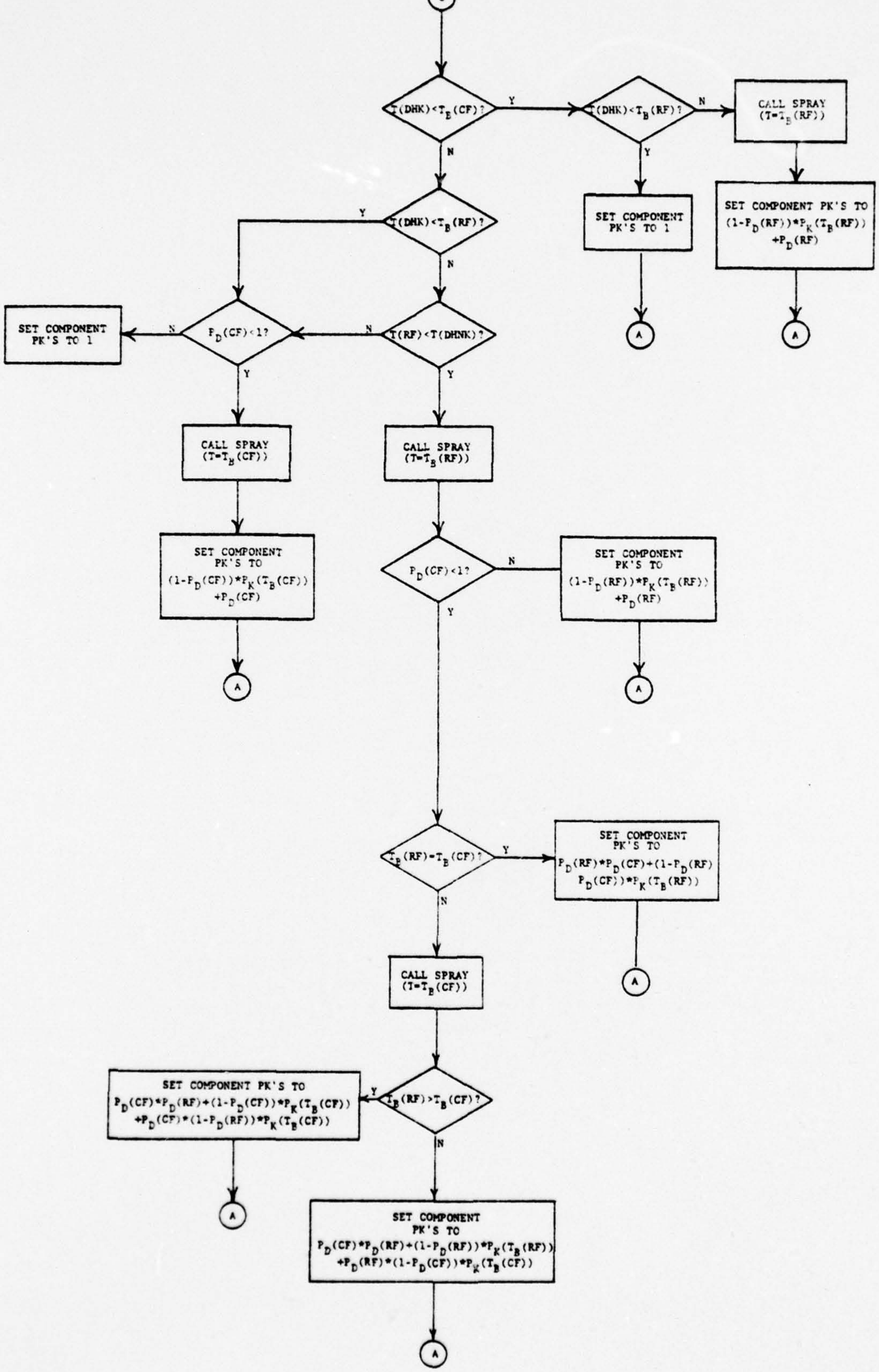


FIGURE 2-1 (continued)




C
T(DHK)<T_B(CF)?
Y
N
T(DHK)<T_B(RF)?
N
CALL SPRAY (T=T_B(RF))
Y
SET COMPONENT PK'S TO 1
SET COMPONENT PK'S TO (1-P_D(RF))*P_K(T_B(RF)) +P_D(RF)
A
A
T(DHK)<T_B(RF)?
Y
N
P_D(CF)<1?
N
SET COMPONENT PK'S TO 1
T(RF)<T(DHNK)?
N
Y
Y
CALL SPRAY (T=T_B(CF))
CALL SPRAY (T=T_B(RF))
SET COMPONENT PK'S TO (1-P_D(CF))*P_K(T_B(CF)) +P_D(CF)
A
P_D(CF)<1?
N
SET COMPONENT PK'S TO (1-P_D(RF))*P_K(T_B(RF)) +P_D(RF)
A
Y
T_B(RF)=T_B(CF)?
Y
SET COMPONENT PK'S TO P_D(RF)*P_D(CF)+(1-P_D(RF) P_D(CF))*P_K(T_B(RF))
A
N
CALL SPRAY (T=T_B(CF))
T_B(RF)>T_B(CF)?
Y
SET COMPONENT PK'S TO P_D(CF)*P_D(RF)+(1-P_D(CF))*P_K(T_B(CF)) +P_D(CF)*(1-P_D(RF))*P_K(T_B(CF))
A
N
SET COMPONENT PK'S TO P_D(CF)*P_D(RF)+(1-P_D(RF))*P_K(T_B(RF)) +P_D(RF)*(1-P_D(CF))*P_K(T_B(CF))
A

The logic takes into account the possibility of the missile possessing either a radar fuze or a contact fuze or both. Each fuze is considered to have a dud probability associated with it. This dud probability would be supplied as an input by the user. Thus, for the case of a missile which has no contact fuze, the contact fuze dud probability would simply be set to 1.0.

An assumption which is implicit in the logic used is that the radar fuze will not be activated after a direct hit has occurred.

2.2 Coordinate Systems and Transformations

In order to avoid any confusion arising from differing sign conventions, the coordinate systems used in the derivations of the endgame analysis were chosen to be identical to those defined in Autonetics' Final Report on the Dynamic SAM-Aircraft Study. The three rectangular coordinate systems employed are:

1) An inertial coordinate system (defined as the "aircraft state vector coordinate system," page 2-10, Volume II; see Figure 4-1, page 4-7, Volume III). This coordinate system has its origin at the SAM site. The Z-axis is perpendicular to the surface of the earth at the SAM site and positive upwards. The X-axis and the Y-axis lie in the plane perpendicular to the Z-axis and are defined so as to create a right-handed system.

2) An aircraft body coordinate system (defined, page 3-28, Volume II). The X-axis of this coordinate system is parallel to the thrust vector of the aircraft. The Y-axis is perpendicular

to the X-axis and positive in the direction of the left wing. The Z-axis is positive upward through the pilot's head. Because in the basic DSAMAM program the aircraft is essentially considered to be a single point, the location of the origin relative to the components of the aircraft is not specified in the definition of the coordinate system. However, since the endgame addition represents the aircraft in a variety of ways (e.g., a set of points or a set of ellipsoids) the location of the origin must be specified for the aircraft body coordinate system. The choice is arbitrary; however, the user will probably wish the point to correspond to some natural feature of the aircraft (e.g., nose, center of gravity, pilot).

3) A missile body coordinate system (defined, page 2-19, Volume III; see Figure 2-6, Volume III). The X-axis in the missile body system corresponds to the longitudinal axis of the missile and is positive towards the nose. The Y-axis is perpendicular to the X-axis and lies within a plane parallel to the inertial XY-plane. The Y-axis is positive to the right of an observer sitting on the missile and facing forward. The Z-axis is perpendicular to the other two and chosen so as to form a right-handed coordinate system. Thus, for a missile flying level, the Z-axis is positive downwards.

In order to facilitate conversion between these coordinate systems, three transformation matrices are computed and stored in COMMON storage. This is done in subroutine MISSIL. These matrices are named TMI, TIA, and TAM.

The first matrix, TMI, is so named because multiplication of a column vector by this matrix will rotate the vector from a coordinate system based on the Missile body axes to the Inertial coordinate system. Similarly, multiplication by TIA rotates a column vector between the Inertial coordinate system and a system based on the Aircraft stability axes, and multiplication by TAM rotates a column vector from the Aircraft system to the Missile system.

These matrices are calculated as follows:

$$\mathrm{TMI} = \begin{pmatrix} \cos(\theta_M)\ \cos(\Psi_M) & -\sin(\theta_M) & \cos(\theta_M)\ \sin(\Psi_M) \\ -\sin(\theta_M)\ \cos(\Psi_M) & -\cos(\theta_M) & -\sin(\theta_M)\ \sin(\Psi_M) \\ \sin(\Psi_M) & 0 & -\cos(\Psi_M) \end{pmatrix}$$

where θ_M and Ψ_M are the elevation and azimuth, respectively, of the missile body axis in the inertial coordinate system (defined, page 2-19, Volume III of the Final Report). Ψ_M is positive when the missile is above the inertial XY plane and θ_M is positive when the positive X-axis in the missile body system is to the right of the positive X-axis in the inertial system.

$$\mathrm{TIA} = \begin{pmatrix} \hat{T} \\ \hat{a}_1 \\ \hat{d}_1 \end{pmatrix}$$

where $\hat{T}$, $\hat{a}_1$, and $\hat{d}_1$ are the unit vectors along the aircraft body axes in the inertial coordinate system (defined, page 3-22, Volume II).

$$TAM = (TMI)^T (TIA)^T ,$$

where the superscript T signifies the transpose of the matrix.

2.3 Fuzing

The endgame model has the capability to simulate a conical pattern radar fuze with a back-up contact fuze. The user supplies as input to the model the dud probabilities for each of these fuzes. Thus, for example, by reading in a dud probability of 1 for the contact fuze, it is possible to simulate a missile with radar fuze only and no back-up.

2.3.1 Conical Radar Fuze

The primary fuzing mechanism is assumed to be a conical pattern radar device. It is simulated in the endgame model by a right circular cone whose apex is at the missile's center of gravity. The fuze cone's axis coincides with the missile's body axis and its half-angle, θ, and range R, are input quantities. If θ is read in as exactly 90°, the simulation is bypassed and fuzing occurs at the point of closest approach of the missile to the target.

In order to analyze the simulation the missile body coordinate system is convenient. Thus, in missile body coordinates the equation of the fuze cone is:

$$Y^2 + Z^2 = X^2 \tan^2\theta .$$

The target is represented by a set of glitter points. These points move relative to the fuze cone in this coordinate system. Let the missile body coordinates of a particular glitter point

at the end of the previous interval be: $P_1:(XG(1),YG(1),ZG(1))$ and its coordinates now be: $P_2:(XG(2),YG(2),ZG(2))$. Then, the line joining these two points is represented by the equation:

$$\frac{XG(1) - X_3}{XG(1) - XG(2)} = \frac{YG(1) - Y_3}{YG(1) - YG(2)} = \frac{ZG(1) - Z_3}{ZG(1) - ZG(2)} = \delta$$

where δ locates any third point, $P_3:(X_3,Y_3,Z_3)$ along the line.

The point P_3 lies between P_1 and P_2 if and only if $0 \leq \delta \leq 1$. Now, for P_3 to lie on the fuze cone, (X_3,Y_3,Z_3) must also satisfy:

$$Y_3^2 + Z_3^2 - X_3^2\tan^2\theta = 0$$

Solving these two equations simultaneously for X_3 results in the quadratic:

$$AX_3^2 + BX_3 + C = 0$$

where,

$$A = (YG(1)-YG(2))^2 + (ZG(1)-ZG(2))^2 - \tan^2\theta\cdot(XG(1)-XG(2))^2$$

$$B = 2\cdot[(YG(2)\cdot XG(1)-YG(1)\cdot XG(2))\cdot(YG(1)-YG(2)) + (ZG(2)\cdot XG(1)-ZG(1)\cdot XG(2))\cdot(ZG(1)-ZG(2))]$$

$$C = (YG(2)\cdot XG(1)-YG(1)\cdot XG(2))^2 + (ZG(2)\cdot XG(1)-ZG(1)\cdot XG(2))^2$$

and therefore,

$$X_3 = \frac{-B\pm\sqrt{B^2 - 4\cdot A\cdot C}}{2\cdot A},$$

If the roots are complex, the line P_1 P_2 doesn't intersect the cone and fuzing does not occur due to this glitter point during this time pulse. If the half-angle of the fuze is less than 90° only positive roots are of interest. If the half-angle is greater than 90° only negative roots are of interest. However,

in order to simplify the computer coding, a check is made at the very beginning of the fuze subroutine and, if the half-angle is greater than 90°, the x-coordinates of the glitter points are multiplied by -1. Because the two nappes of a cone are symmetrical, this has the effect of requiring all roots of interest to be positive.

The root resulting from the plus sign of the radical represents the earlier intersection. This root is calculated first and tested to see if it lies between 0 and $R \cdot \cos\theta$. If it is negative then the other root must also be negative and fuzing does not occur due to this glitter point for this time pulse. If it is positive but greater than $R \cdot \cos\theta$, then the other root is tested. If the root does lie between 0 and $R \cdot \cos\theta$, then δ is calculated from the relationship:

$$\delta = \frac{XG(1) - X_3}{XG(1) - XG(2)} .$$

If this quantity is greater than 1 then it must be greater than 1 for the other root also, and fuzing does not occur due to this glitter point for this time pulse. If δ is negative, then the other root is examined. If $0 \leq \delta \leq 1$, then fuzing does occur due to this glitter point for this time pulse.

All glitter points are tested in the above fashion. If none of them result in a value of δ between 0 and 1, then fuzing does not occur for the time pulse. Otherwise, the minimum non-negative δ is used to determine the time of fuzing.

2.3.2 Contact Fuze

In order to simulate the back-up contact fuze, the aircraft is represented by a set of direct-hit-without-kill ellipsoids (which represent the skin of the aircraft). If the missile trajectory intersects any of these ellipsoids before the warhead has detonated, then the contact fuze is initiated. Thus, for each of the direct-hit-without-kill ellipsoids the following system of equiations in aircraft body coordinates must be solved.

Let the prior position of the missile be: P1:(XC(1),YC(1),ZC(1)), its current position be: P2:(XC(2),YC(2),ZC(2)), and a third point along the trajectory be: P3:(X_3,Y_3,Z_3). If P3 is also on the ellipsoid, it must simultaneously satisfy:

$$\frac{XC(1)-X_3}{XC(1)-XC(2)} = \frac{YC(1)-Y_3}{YC(1)-YC(2)} = \frac{ZC(1)-Z_3}{ZC(1)-ZC(2)} = \delta$$

and,

$$\frac{(X_3-XO)^2}{(XS)^2} + \frac{(Y_3-YO)^2}{(YS)^2} + \frac{(Z_3-ZO)^2}{(ZS)^2} = 1$$

where XS, YS, and ZS are the three semi-axes of the ellipsoid and (XO,YO,ZO) is the center of the ellipsoid.

Solving this system for Z_3 results in the quadratic, where:

$$\begin{aligned} A = {} & [(ZC(1)-ZC(2))\cdot XS\cdot YS]^2+[(YC(1)-YC(2))\cdot XS\cdot ZS]^2+ \\ & [XC(1)-XC(2))\cdot YS\cdot ZS]^2 \end{aligned}$$

$$\begin{aligned} B = -2\cdot\{ & (XC(1)-XC(2))\cdot YS^2\cdot ZS^2\cdot[XC(1)\cdot ZC(2)-XC(2)\cdot ZC(1)+ \\ & \qquad XO\cdot(ZC(1)-ZC(2))]+ \\ & (YC(1)-YC(2))\cdot XS^2\cdot ZS^2\cdot[YC(1)\cdot ZC(2)-YC(2)\cdot ZC(1)+ \\ & \qquad YO\cdot(ZC(1)-ZC(2))]+ \\ & ZO\cdot[XS\cdot YS\cdot(ZC(1)-ZC(2))]^2\} \end{aligned}$$

$$C = \{XS^2 \cdot ZS^2 \cdot [YC(1) \cdot ZC(2) - YC(2) \cdot ZC(1) + YO \cdot (ZC(1) - ZC(2))]^2 + YS^2 \cdot ZS^2 \cdot [XC(1) \cdot ZC(2) - XC(2) \cdot ZC(1) + XO \cdot (ZC(1) - ZC(2))]^2 + XS^2 \cdot YS^2 \cdot (ZC(1) - ZC(2))^2 \cdot (ZO^2 - ZS^2)\}$$

and therefore,

$$Z_3 = \frac{-B \pm \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A}$$

If the roots are complex, no direct hit occurs on this ellipsoid for this time pulse. If real roots do exist, the root which represents the earlier intersection is the root arising from the case where the radical has the same sign as (ZC(1)-ZC(2)). This root, therefore, is the only one that is examined. The quantity δ is calculated from the relationship:

$$\delta = \frac{ZC(1) - Z_3}{ZC(1) - ZC(2)}$$

If $0 \leq \delta \leq 1$, then P_3 lies between P_1 and P_2, and the solution represents a valid direct hit for the current time pulse.

All direct-hit-without-kill ellipsoids are tested in this manner and the solution which gives rise to the least positive δ is used to calculate the time at which the contact fuze is initiated.

2.4 Single-Shot Component Kill

Three kill mechanisms are considered in the endgame model. These are direct-hit kill, blast kill and fragment kill. Direct-hit kill occurs whenever the as yet undetonated missile strikes the aircraft in such a way as to cause a kill even without the warhead effects. Blast kill results from the shock wave formed

in the surrounding atmosphere upon detonation of the explosive warhead. This shock wave can damage an aircraft at close range by overpressure and impulse loading. Fragment kill occurs when fragments of the warhead casing, propelled by the force of the warhead detonation, intercept one or more of the vulnerable components of the aircraft.

2.4.1 Direct-Hit Kill

Direct-hit kill is simulated in the endgame model by representing the target as a set of direct-hit-with-kill ellipsoids. The process of determining whether or not such a direct hit occurs in particular time pulse is identical to the process described in Section 2.3.2 except that the direct-hit-with-kill ellipsoids are used in place of the direct-hit-without-kill ellipsoids.

2.4.2 Blast Kill

A balst kill occurs whenever the burst point (X*,Y*,Z*) lies within one or more of a set of blast ellipsoids. The equation expressing this condition in aircraft body coordinates is:

$$\frac{(X^*-XBL)^2}{(ABL)^2} + \frac{(Y^*-YBL)^2}{(BBL)^2} + \frac{(Z^*-ZBL)^2}{(CBL)^2} \leq 1,$$

where (XBL,YBL,ZBL) is the center point of a blast ellipsoid and ABL, BBL and CBL are the scaled semi-axes lengths for that ellipsoid. This test is performed for all blast ellipsoids and blast kill is said to occur if it is satisfied for any one of them.

2.4.3 Fragment Kill

The static explosion of a fragment warhead produces a characteristic spectrum of fragment mass, fragment density and fragment

emission speed. The explosion of a miving fragment warhead alters this spectrum by virtue of the imposed forward speed of the missile. It is necessary to determine the interaction of this altered spectrum with the moving aircraft in order to obtain expressions for the expected number of fragment hits upon the vulnerable components of the aircraft and for the net striking speed of these fragments.

The distance-time equation governing the fragments can be written:

$$\ln(1+C_D \cdot m^{-1/3} \cdot \rho \cdot V_0 \cdot t) = C_D \cdot m^{-1/3} \cdot \rho \cdot L$$

where, C_D is the sea level drag coefficient for the fragment, m is the fragment mass, ρ is the relative air density, V_0 is the initial fragment speed, t is the time of flight of the fragment and L is the distance travelled by the fragment. This equation has no closed form solution. The endgame program arrives at a solution using the Newton-Raphson iterative method to find a root of the equation:

$$F(t) = \ln(1+C_D \cdot m^{-1/3} \cdot \rho \cdot V_0 \cdot t) - C_D \cdot m^{-1/3} \cdot \rho \cdot L = 0$$

This classical method takes the Jth estimate of the root, $t^{(J)}$, and extracts the (J+1)th estimate, $t^{(J+1)}$, by means of the following equation:

$$t^{(J+1)} = t^{(J)} - \frac{F(t^{(J)})}{\dot{F}(t^{(J)})}$$

The procedure is repeated until successive estimates are considered to differ neglibibly.

As a first estimate of the solution is used the time of flight resulting from the case where the fragment does not slow down due to drag. The solution for the fragment time of flight and distance travelled also determines the dynamic emission angle, γ, between the missile velocity vector, $\vec{V}_M$, and the dynamic fragment velocity vector, $\vec{V}_O$. This in turn determines the angle θ, between the static fragment velocity vector, $\vec{V}_E$, and $\vec{V}_M$ (see Figure 2-2).

As a result of the rotation of velocity vectors due to the missile motion, the fragment density in the static case, Ψ, is unequal to the dynamic density, Ψ_{DYN}:

$$\Psi_{DYN} = \Psi/E \ .$$

The term E can be shown to be:

$$E = \frac{\sin\gamma}{\sin\theta} \cdot \frac{d\gamma}{d\theta}$$

$$= \frac{A^2 \cdot |A+\cos\theta|}{[A^2+2 \cdot A \cdot \cos\theta+1]^{3/2}}$$

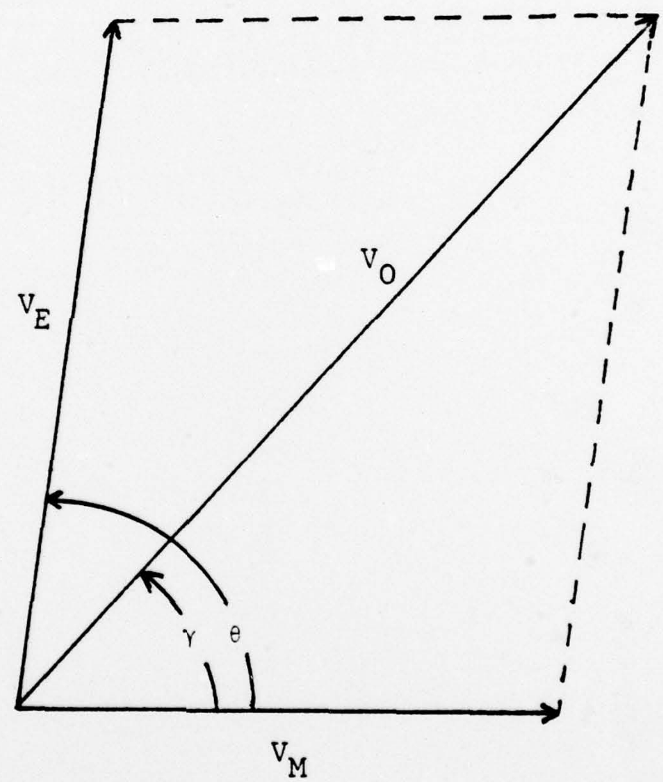
where,

$$A = V_E/V_M$$

Input data exist for the fragment density resulting from a static explosion in the form of a table having entries at 10^o intervals off the nose of the missile. Thus, in order to interpolate for ψ from the input table, it is necessary to find the angle, θ_B, between V_E and the nose of the missile. This is accomplished by transforming the vector $\vec{V}_E$ (= $\vec{V}_O$ - $\vec{V}_M$)

FIGURE 2-2

STATIC AND DYNAMIC FRAGMENT EMISSION



into missile body coordinates using the matrix TAM and taking the dot product of this with the vector (1,0,0).

In the Dynamic SAM-Aircraft Model the aircraft velocity vector, $\vec{V}_T$, is not necessarily parallel to the aircraft body axis. Thus, in the aircraft body coordinate system:

$$\vec{V}_T = V_T(1)\cdot\vec{i} + V_T(2)\cdot\vec{j} + V_T(3)\cdot\vec{k}$$

Consider now a coordinate system which is identical to the aircraft body system at the time of detonation but which remains fixed in space. Let (X_0, Y_0, Z_0) be the coordinates of one of the vulnerable components of the aircraft in this stationary system at the time of detonation. Then, if TF is the time of flight of the fragment, the coordinates of this component at the time it is hit will be:

$$X_H = X_0 + V_T(1)\ TF$$

$$Y_H = Y_0 + V_T(2)\ TF$$

$$Z_H = Z_0 + V_T(3)\ TF$$

As previously defined in Section 2.4.2, (X^*, Y^*, Z^*) represents the coordinates of the burst point in the aircraft body system at the time of detonation. Therefore,

$$L = \sqrt{(X_H - X^*)^2 + (Y_H - Y^*)^2 + (Z_H - Z^*)^2}$$

and the direction cosines, with respect to the aircraft body system axes, of the line directed from the explosion point to the hit point are β_X, β_Y and β_Z:

$$\beta_X = \frac{X_H - X^*}{L}$$

$$\beta_Y = \frac{Y_H - Y^*}{L}$$

$$\beta_Z = \frac{Z_H - Z^*}{L}$$

The above method of calculating hits assumes that there are no masks or shields to stop the fragments before they reach the vulnerable component. Shielding is incorporated into the model by the approximate but adequate treatment described below.

The hit-point location of a vulnerable component is calculated, assuming that the shield is non-existent. This defines the time of flight of the fragment, TF, and also the straight line path of the fragment (from explosion point to hit point). The shield is now placed in the position it would occupy at the time of the hit. The shield is taken to be an ellipsoid whose axes are parallel to the X, Y, Z axes of the aircraft body co-ordinate system. If the fragment travel line intersects the shield, the vulnerable component is considered to be shielded. If there is no intersection, the component is not shielded.

This treatment should be accurate if, at the time of the explosion, the shield is much closer to the component than to the explosion point. A higher level of accuracy would involve ballistic solutions for fragment and shield. This appears both complicated and unwarranted.

The speed of the fragment at the time it strikes the component, V_{HIT}, is given by:

$$V_{HIT} = V_0 \cdot e^{-C_D \cdot m^{-1/3} \cdot \rho \cdot L}$$

Therefore, in the aircraft body system:

$$\overrightarrow{V_{HIT}} = V_{HIT} \cdot \beta_X \cdot \vec{i} + V_{HIT} \cdot \beta_Y \cdot \vec{j} + V_{HIT} \cdot \beta_Z \cdot \vec{k}$$

and the relative velocity vector between the fragment and the aircraft, $\overrightarrow{V_{NET}}$, is:

$$\begin{aligned}\overrightarrow{V_{NET}} &= \overrightarrow{V_{HIT}} - \overrightarrow{V_T} \\ &= (V_{HIT} \cdot \beta_X - V_T(1)) \cdot \vec{i} + (V_{HIT} \cdot \beta_Y - V_T(2)) \cdot \vec{j} \\ &\quad + (V_{HIT} \cdot \beta_Z - V_T(3)) \cdot \vec{k}\end{aligned}$$

The net striking speed, V_{NET}, is the magnitude of the vector $\overrightarrow{V_{NET}}$:

$$V_{NET} = \sqrt{(V_{HIT}\ \beta_X - V_T(1))^2 + (V_{HIT}\ \beta_Y - V_T(2))^2 + (V_{HIT}\ \beta_Z - V_T(3))^2}$$

Fragments approaching a target component along a given line can strike at most three of its primary orthogonal aspects. The signs of the components of $\overrightarrow{V_{NET}}$ identify the aspects struck as shown in Table 2.1 below.

TABLE 2.1

CRITERIA FOR ASPECTS STRUCK

Criterion	Aspect Struck
$\beta_X > \frac{V_T(1)}{V_{HIT}}$	Rear
$\beta_X < \frac{V_T(1)}{V_{HIT}}$	Front
$\beta_Y > \frac{V_T(2)}{V_{HIT}}$	Right
$\beta_Y < \frac{V_T(2)}{V_{HIT}}$	Left
$\beta_Z > \frac{V_T(3)}{V_{HIT}}$	Bottom
$\beta_Z < \frac{V_T(3)}{V_{HIT}}$	Top

To calculate the number of fragment hits on the component, it is necessary to know the fragment density along the vector $\overrightarrow{V_{NET}}$. It appears to be very difficult mathematically to arrive at this density, however, Therefore, the approximation is made that the number of hits can be calculated on a static target (equivalent to using the density ψ_{DYN}, along the fragment velocity vector). This is an excellent approximation if $V_{HIT} >> V_T$. In situations in which this condition does not

hold, the striking energy of a fragment would be so low as not to cause an appreciable probability of kill.

Using the static target concept, then, Table 2.1 is modified, replacing the terms $V_T(1)/V_{HIT}$, $V_T(2)/V_{HIT}$ and $V_T(3)/V_{HIT}$ by zero. The vulnerable areas of the relevant aspects are projected onto a plane perpendicular to the fragment velocity vector in the following manner:

$$\overline{VA} = |\beta_X| \cdot VA_X + |\beta_Y| \cdot VA_Y + |\beta_Z| \cdot VA_Z$$

The expected number of lethal hits on the component, N, can now be calculated:

$$N = \psi_{DYN} \cdot \overline{VA}/L^2$$

Applying Poisson's Law gives an expression for the probability of survival of the component:

$$PS = e^{-N}$$

The above calculations are carried out in the endgame program for all components and at all kill levels.

2.5 Overall Aircraft Kill

The single-shot aircraft kill probability, PKAC, is calculated from the following relationship for an aircraft composed only of singly vulnerable components.

$$PKAC = 1 - \prod_{i=1}^{NCOMPS} PS_i$$

where PS_i is the survival probability of component i and NCOMPS is the total number of vulnerable components.

This formulation is altered somewhat when there are one or more pairs of doubly vulnerable components. If j and k form a pair of doubly vulnerable components, then the term $PS_j \cdot PS_k$ is replaced in the above product by the probability that both j and k are not killed: $[1-(1-PS_j)\cdot(1-PS_k)]$.

In order to calculate the cumulative aircraft kill probability resulting from a salvo of missiles it is necessary to compute and save the cumulative survival probability, CPS_i, for each vulnerable component, i:

$$CPS_i = \prod_{\substack{\text{all} \\ \text{shots}}} PS_i$$

The cumulative aircraft kill then becomes

$$CPKAC = 1 - \prod_{i=1}^{NCOMPS} CPS_i$$

with $CPS_j \cdot CPS_k$ being replaced by $[1-(1-CPS_j)\cdot(1-CPS_k)]$ for any pair, j and k, of doubly vulnerable components.